

The Marist Catholic Primary School


Progression in Calculation

## Welcome

- Have a go at some Year 6 Arithmetic questions!


## The Aims Of The Evening

- Share with you the ways in which calculations (the four rules of addition, subtraction, multiplication and division) are taught at the Marist
- Share with you the expectations in Maths at Year 6
- Give you some tips on how to support your child in their learning of Maths


## By The End Of Key Stage 2

- What is the problem about - which operations (,,$+- x$ or $\div$ ) will I need?
- Can I do this in my head?
- Can I do it in my head but with the help of some drawings or jottings?
- Do I need to use a written method?
- And then, having completed the calculation...
- Is my answer a reasonable or sensible one?


## Progression in Addition

Number Names and ordering
One of the first things we do is teach the children the names of the numbers and their order. This means that the children can begin to count objects and to see the relationship between how many and an actual number.

Some number relationships will be taught such as the "number bonds to $10^{\prime \prime}$. This is important for the children to understand the effect of adding a 3 to a 7 or a 1 to a 9 and so on.


## Number lines and other graphic methods

The next step is to use numbers of objects to add For example, if I eat 3 sweets and then 2 more, how many have I eaten?


The children can use actual groups of objects to work out the answer once their understanding of the number names and order is secure.
The children may then start independently to draw pictures or make a tally to help them with the addition.

This progresses to the use of a number line:


From here the children can continue to add by counting on. So, if I eat 3 more sweets, how many will I have eaten in total?

Please note that the use of the number line continues right through the school - it is not an infants' tool! An "empty number line"can be used for more complicated calculations such as $457+59$, provided that the child has a good understanding of number and can "partition" ie see 59 as $50+9$


## Partitioning and recombining

As the calculations get more complicated and the use of mental maths on its own or number lines become less efficient, so the children progress to more formal written methods.

These rely on an understanding of place value and the ability to partition ie knowing that the digit 4 in 46 actually has a value of 40 or 4 tens etc, meaning that 46 is made up of $40+6$

A calculation can then be completed as follows:
My sunflower is 47 cm tall. It grows another 39 cm . How tall is it now?
Tens Units
$40+6$
$30+9$
$70+15=85$

## Expanded written method with exchanging

There is then a natural progression to start to talk about "exchange" where, for example, 10 units are exchanged as 1 ten and shown in the tens column:

There are 467 girls and 529 boys in the secondary school. How many children all together?

$$
\begin{aligned}
& \text { Hundreds Tens Units } \\
& \begin{array}{l}
400+60+7 \\
\frac{500+20+9}{900+90+6} \\
10
\end{array}=996
\end{aligned}
$$

## Compact written methods

Once children are confident with the expanded written method, then they are ready for the more traditional compact method of addition: there is nothing to be gained - except confusion - by teaching this method until children have a real and secure understanding of place value and of the concept of exchange. This is generally in upper Key Stage 2.

6,874 Woking supporters were at the Final and 3,837 Luton fans. How many fans were there in total?

$$
\begin{array}{r}
68774+ \\
38837 \\
100711 \\
\hline 1111
\end{array}
$$

## Progression in Subtraction

Number Bonds
Again, simple subtraction can follow an understanding of basic number bonds to 10 . If I know $3+7$ is 10 , the next step of understanding is that if I take 3 away from 10 , I will be left with 7.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

$$
7-3
$$



## Practical and Graphic Methods

Children will start by using objects as soon as they can reliably count and identify the order of number. For example, I have 5 balloons but 2 burst. How many have I got left

5 take away 2 ... there are 3 left


Working independently the children will then start to show the calculation by drawing or using dots or a tally - it shows that they can visualise the calculation: the first step to a mental calculation.
This will then progress to the use of the number line.

## Counting back and counting on

As with addition, once the children are confident with place value and partitioning, they can then start to use the empty number line efficiently
Counting back:
I cut 27 cm from a ribbon measuring 84 cm . How much is left?


## Counting on:

Sometimes it is easier to count on from the smaller number to the larger number to find the difference between them.

The library lends 378 of its 834 books. How many are left on the shelves?

The calculation is $834-378$ or finding the difference between 378 and 834...


The first written method introduced is an expanded one to help the children understand what is happening. It is at this stage that the children need to start exchanging tens for ten units or hundreds for 10 tens. Traditionally we have may said "borrowed" but this gives a false impression that we will give it back! Today we use the term "exchange".
There are 43 children in the hall and 27 leave. How many children are still in the hall?


Here is another example where a 10 is exchanged from the tens column to the units AND a 100 is exchanged from the hundreds column to the tens.
There are 754 ants in the ant hill but 286 leave to collect food. How many ants are left in the ant hill?

| Hundreds | Tens | Units |
| :--- | ---: | ---: |
| 600700 | $100+4050$ | $10+4$ |
| $-\quad 200$ | 80 | 6 |
| 400 | and | 60 |
|  | and | 8 |

## Compact written methods with exchangingsins

Once children are confident with what they are doing, then it is possible to compact the method to the more traditional written method with which parents may be more familiar. Again, without an understanding of why they are doing this, the method can be very confusing indeed!
There are 643 children in the hall and 287 leave. How many children are still in the hall?


## Progression in Multiplication

Repeated addition with graphic methods
Children are introduced to multiplication first as repeated addition. As in every case this will start with the use of physical objects and then move to pictures and other graphic methods. For example:
Each child has two feet. How many feet do four children have?


## Arrays and Number Lines

An array is a way of showing data in a regular rectangular form of columns and rows so, for example it can help with $3 \times 4$ :


This also helps children to understand that $3 \times 4$ is the same as $4 x$ 3.

The use of the number line is for repeated addition: There are 4 cats. Each cat has 6 kittens. How many cats are there altogether?


This can be developed for larger numbers by using known facts: There are 13 biscuits in a packet. How many biscuits in 7 packets?


## Grid method with partitioning

The more advanced use of the number line leads naturally to the grid method - the first written method we teach for multiplication. This is dependent on a secure understanding of place value and partitioning.

|  | 10 | 3 |
| :---: | :---: | :---: |
| 4 | $40+$ | 12 |

A block is 193 cm high. How high would 6 blocks be if they were placed on top of each other?

| $x$ | 100 | 90 | 3 |
| :---: | :---: | :---: | :---: |
| 6 | $600+$ | $540+$ | 18 |

This method can be used for numbers of any size provided the child is confident in partitioning:
Eg $34 \times 27$

| $x$ | 30 | 4 |  |
| :---: | :---: | :---: | :--- |
| 20 | $600+$ | 80 | 680 |
| 7 | $210+$ | 28 | $238+$ |
| so | $34 \times 27=$ |  | 918 |

## Expanded written method

This method takes the grid method and starts to put it in to a vertical structure as an introduction to the conventional compact method. Again, it depends on a secure understanding of place value and partitioning:
A block is 193 cm high. How high would 6 blocks be if they were placed on top of each other?
$193 \times 6$

| 93 |  |
| ---: | :--- |
| $\times \quad 6$ |  |
| 18 | $(3 \times 6)$ |
| 540 | $(90 \times 6)$ |
| 600 | $(100 \times 6)$ |
| 1158 |  |

## Compact Written Method

The expanded method is eventually reduced to the compact method for short multiplication (a number $x$ by a single digit). The recording is reduced with the carry digits recorded below the line.

| 19 | 3 |
| ---: | :--- |
| $\times$ | 6 |
| 1 | 15 |
| 151 |  |

## Long Multiplication

The final step is the introduction of long multiplication (a number multiplied by more than 1 digit).
Children can use the grid method, the expanded method or the compact method.
However, it is essential that children understand place value before tackling the compact method of long multiplication as they have to understand the need to place a 0 in the units column when multiplying by tens in order to "hold the place", to be a placeholder...
There are 52 weeks in a year. How many weeks in 27 years? 52 x 27

There are 52 weeks in a year. How many weeks in 27 years?

| $x$ | 50 | 2 |  |
| :---: | :---: | :---: | :---: |
| 20 | $1000+$ | 40 | 1040 |
| 7 |  |  |  |
| so | $350+$ | 14 | $364+$ |
| $52 \times 27=$ |  |  |  |
|  |  |  |  |


| $\begin{array}{r} 52 \\ \times 27 \\ \hline \end{array}$ |  |  | $(7 \times 2)$ |
| :---: | :---: | :---: | :---: |
|  |  |  |  |
|  | 1 | 4 |  |
| 3 | 5 | 0 | $(7 \times 50)$ |
|  | 4 | 0 | $(20 \times 2)$ |
| 10 | 0 |  | $(20 \times 50)$ |
| 14 | 40 |  |  |


|  | 52 |  |
| ---: | ---: | :--- |
| $\times 27$ |  |  |
|  | 364 | $(52 \times 7)$ |
| +10400 | $(52 \times 20)$ |  |
| 14044 |  |  |

## Progression in Division

Division is seen as the hardest operation - but it shouldn't be! It is the inverse (opposite) to multiplication. A good grasp of times tables, understanding not just reciting, will really help!

## Sharing and grouping

Children start by sharing and grouping.
Again, practical objects and then images are the way in.

For example, There are 6 lollies to be shared between 2 children, How many lollies will each child get?
Sharing: one for you, one for me... until all are gone


Or There are 6 lollies. How many children can have 2 each?
Grouping: in two's


## Repeated Subtraction and Number Lines

Number lines can be used for repeated subtraction:
A lolly costs 7 p. How many can I buy with 28p?


4 jumps of 7 , so $28 \div 7=4$
This can also work where there are remainders.
This can be further developed by using larger "chunks" of the divisor, for example:
I need 6 pins to put up a picture. How many pictures can I put up with 84 pins? $84 \div 6$


So $84 \div 6=14$

## Repeated Subtraction and 'Chunking'

Chunking is simply repeated subtraction of "chunks" of the divisor. However, children need to understand that chunking is inefficient if too many subtractions have to be carried out. The chunks need to be sensible amounts eg a multiple of 10 ...

I have 200 chairs to arrange in to rows of 6 . How many rows will I have?

## Chunking

6) 200
7) 200
$\begin{array}{r}200 \\ -6 \\ \hline 194 \\ -6 \\ \hline 188 \\ -6 \\ \hline\end{array}$
continuing
$-\frac{6}{2}$
33 r 2

200
$\frac{-60}{140}(\underline{10} \times 6$
$\frac{-60}{80}(\underline{10} \times 6$
$\frac{-60}{20}(\underline{10} \times 6$
$\frac{-18}{2}(\underline{3} \times 6)$
33 r 2
6) 200

200
$\frac{-180}{20}(\underline{30} \times 6)$
$\frac{-18}{2}(\underline{3} \times 6)$
$33 r 2$
The more confident the understanding, the larger the 'chunks' and the more efficient the method.

## Short Division

Short division uses the 'bus shelter' method. e.g. I have 200 chairs to arrange in to rows of 8 . How many rows will I have?

$$
\begin{array}{r}
3 \quad 3 \\
6 \quad 0 \quad 0
\end{array}
$$

## Long Division

The final step is to tackle long division.
This is where you divide by more than 1 digit.
For UKS2 children this usually means dividing by a 2 digit number.

Again, depending on their understanding, children can use chunking or the compact method.

## Long Division



It is important that children always try to estimate first to check that their answer makes sense.

Bar Model

## Small steps



$$
4+2=6
$$

## KS2 barmodelling

$\frac{3}{5}$ of $20=$ ?

## Have a

On Saturday Lara read $\frac{2}{5}$ of her book.

On Sunday she read the other 90 pages to finish the book.


How many pages are there in Lara's book?

## How to support your children at home

- TTRockstars - (soundcheck mimics year

4 timestable test)

- Times tables
- Mymaths
- Money and time
- Measurement
- Look at the calculation policy on the
school website.


## Survey

